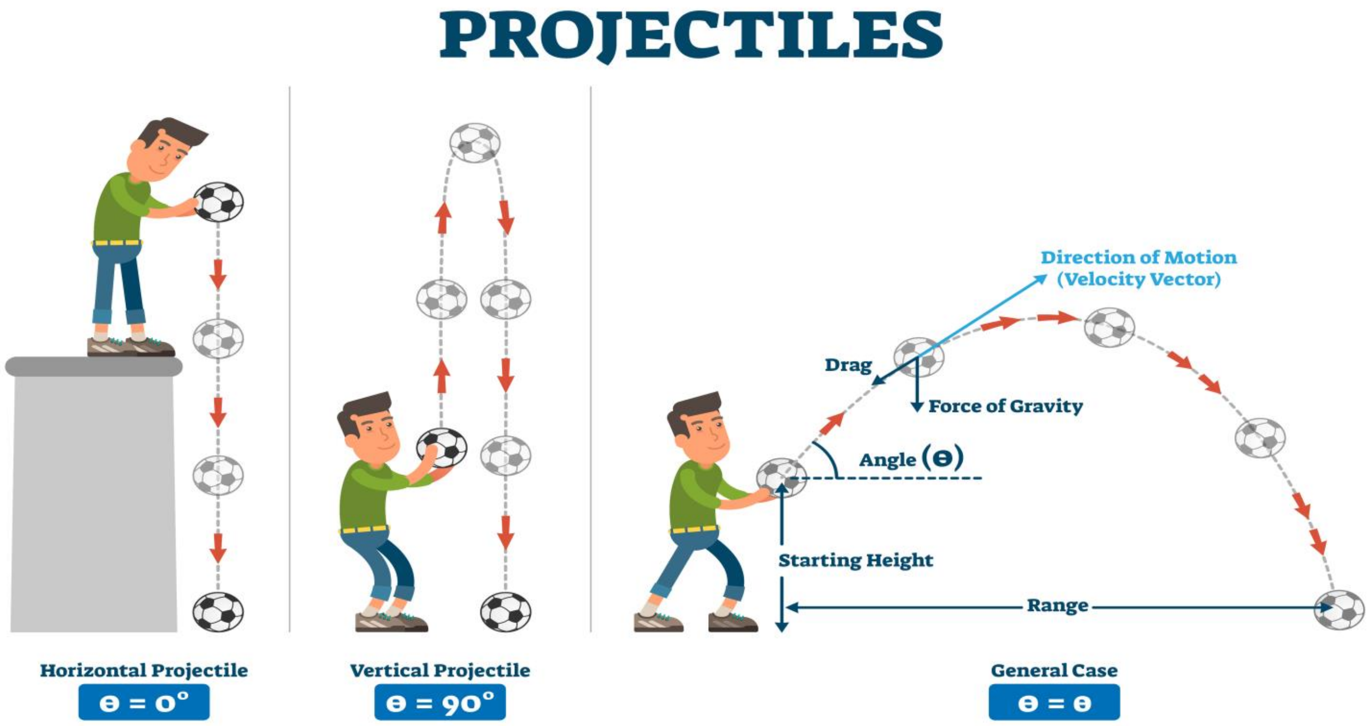


PROJECTILE AND MOTION IN 2-D

NJ-OER TOPIC-3

Learning Outcomes

- Observe that motion in vertical direction on Earth
- Understand the independence of horizontal and vertical components in two-dimensional motion
- Understand the rules of vector addition and subtraction
- Apply graphical and analytical methods of vector addition
- Understand different vector representations and their relationship such as magnitude/angle; component form, graphical representation, vector representation
- Identify, explain and determine range, maximum height, time for fall, velocity and trajectory of a projectile.
- Apply the principle of independence of motion to solve general 2-D motion problems.
- Apply principles of vector addition to determine relative velocity and explain the significance of the observer.

Concepts

- Δx = displacement in x
- Δy = displacement in y
- vo= initial velocity
- vox = initial velocity in x
- voy = initial velocity in y
- vfx =final velocity in x
- vfy = final velocity in x
- t = time or duration
- g= gravitational acceleration
- ax = acceleration in x
- ay = acceleration in y
- VAB=Velocity of object A with respect to observer B
- θ = angle measured from the +x-axis

Units

Position, displacement, distance are in meters “m”
Velocity and speed are in meters per second “m/s”
Acceleration is in meters per second square “m/s^2”
Angle in degrees or radian

Formulas

$\Delta x = v_{ox}\, t + \frac{1}{2} a_x t^2$
$v_{fx} = v_{ox} + a_x t$
$v_{fx}^2 - v_{ox}^2 = 2a_x \Delta x_a$
$v_{ox} = v_o \cos(\theta)$
$v_{oy} = v_o \sin(\theta)$
$a_x = a \cos(\theta)$
$a_y = a \sin(\theta)$
$v^2 = v_x^2 + v_y^2$
$a^2 = a_x^2 + a_y^2$
$\tan\theta = v_y / v_x$

$\Delta y = v_{oy} t + \frac{1}{2} a_y t^2$
$v_{fy} = v_{oy} + a_y t$
$v_{fy}^2 - v_{oy}^2 = 2a_y \Delta y$

Free fall or projectile
$v_{fx} = v_{ox}$
$a_x = 0$
$a_y = -g$
$g = 9.8\ m/s^2$

Relative velocity
Vab = **Vao** + **Vob**

KEY STRATEGIES

Draw the motion diagram, identify given quantities and the unknown
For 2-D problems use trigonometry to identify components
Substitute the known quantities and solve for the unknown using algebra

Maximum height happens at the time when vfy=0. Set vfy to zero, solve for t
Time for the fall is calculated using y=0 or (yf-yi)=-(initial height)
Range is calculated in the x-axis at the time when y=0
Impact speed can be calculated using the timeless equation of kinematics

Released means vo=0

MODEL PROBLEM

Q) An object is projected upward from the ground with 15.0 m/s speed.

A) When will it reach the maximum height

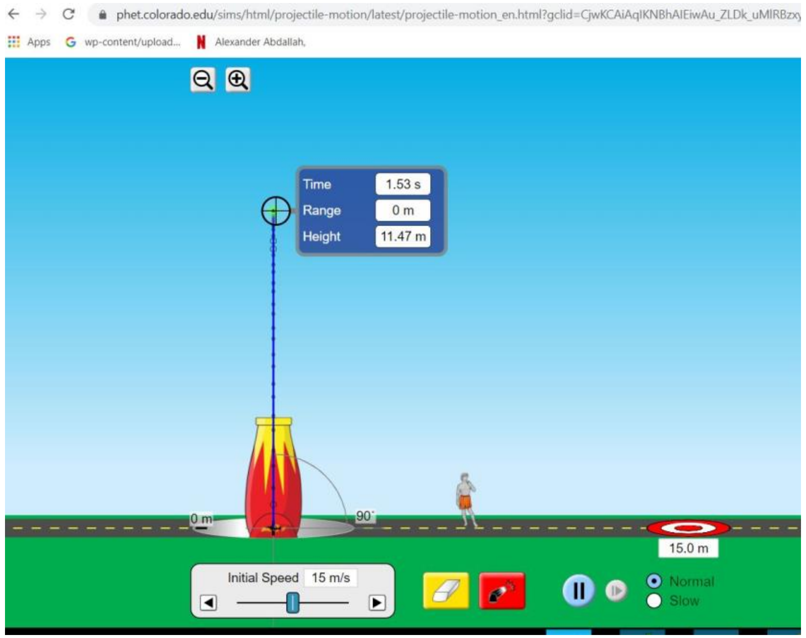
B) What is the maximum height

C) If the object continues its motion after reaching the y-max, what is the impact velocity and what is the time for fall.

ACTIVITY PROJECTILE

Q) An object is projected upward from the ground with 15.0 m/s speed. A) When will it reach the maximum height? B) What is the maximum height?

This problem can be simulated using an online activity. Run the app, obtain the results from the app and compare it with your calculations.

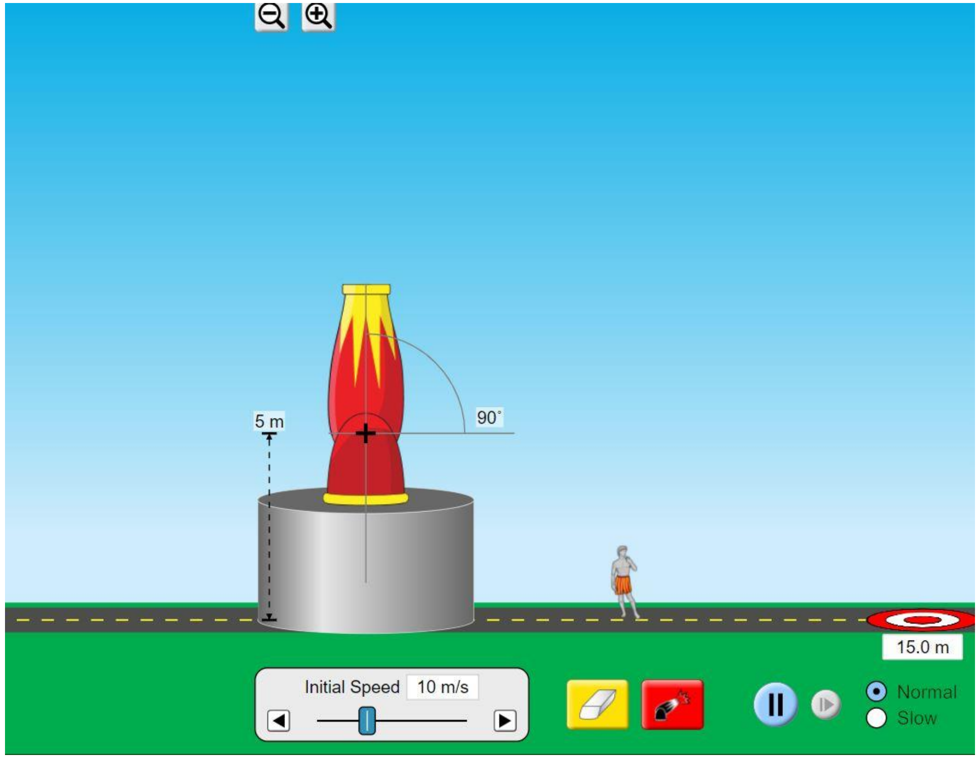


- Open Phet Projectile simulation https://phet.colorado.edu/en/simulations/projectile-motion
- Choose intro or lab. Set the initial height to zero and initial speed to 15m/s.
- Hit the red button and launch the projectile
- Using the probe on the top right, next to the initial values, measure the maximum height and the time for the projectile to reach the maximum height. Compare it with your calculations

ACTIVITY PROJECTILE

Q) An object is projected upward with 10.0 m/s speed from 5 meters height. A) When will it reach the maximum height? B) What is the maximum height? C) Find the impact velocity using timeless equation of kinematics. Using Vfy equation find the total time for fall. D) Obtain the height at t=0.2s by substituting t=0.2 in Δy equation

Run the app, obtain the results from the app and compare it with your calculations.

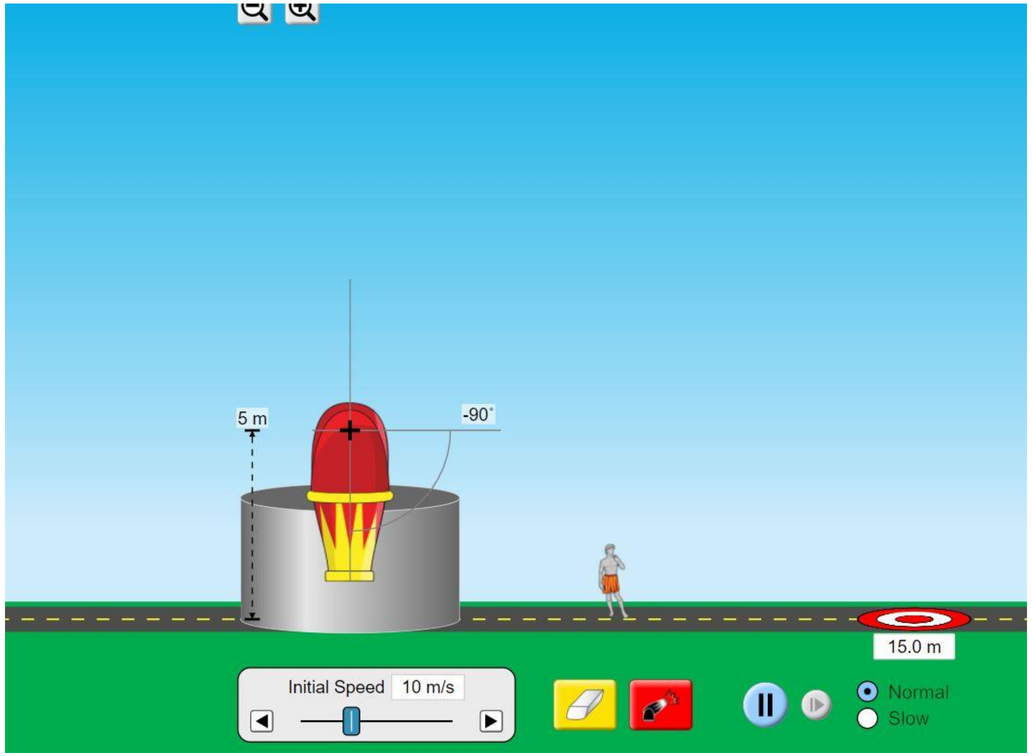


- IMPORTANT NOTES:
- Kinematics equation gives ΔY. You need to add the initial height in order to find ymax
- For the impact velocity, Vfy is always negative

ACTIVITY DOWNWARD PROJECTILE

Q) An object is projected DOWNWARD with 10.0 m/s speed (viy=-10m/s) from 5 meters height. A) What is the maximum height? C) Find the impact velocity using timeless equation of kinematics. Using Vfy equation find the total time for fall. D) Obtain the height at t=0.2s by substituting t=0.2 in Δy equation

Run the app, obtain the results from the app and compare it with your calculations.



- IMPORTANT NOTES:
- For downward projectile, ymax is the original height
- For all the downward projectile viy is negative. Slider tool gives the speed which is the magnitude.

ACTIVITY

Using the given initial values run the simulation. Obtain the results using the probe. Using equation of kinematics calculate the values and compare

Yi(m)	Viy(m/s)	Ymax(m)	Time for Ymax (s)	Time for fall (s)	Ymax (calculated)	Time for ymax (calculated)	Time for fall calculated(s)
5	9						
6	-8						
0	10						
11	0						
11	2						
11	-2						

REVERSE PROBLEMS, FINAL IS GIVEN INITIAL IS ASKED

Q) An object is projected upward from a tower. It reaches ymax=20 meters at t=2.0 seconds.

A) What was the initial velocity viy?

B) What was the initial height.

C) If it continues the motion, what would be the impact velocity, and time for fall

Do the same problem model using different given numbers
1) ymax = 20m t=1.5s Find Viy and the height
2) ymax = 10m t=2.5s Find Viy and the height (this is a projectile from a water well
3) ymax=? t=3 seconds h=12 meters find viy and ymax

VECTOR REPRESENTATIONS 2-D Motion

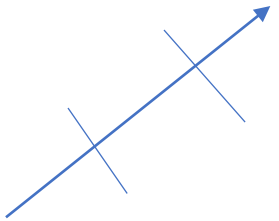
Graphical | Magnitude/Angle | Components | Vector Representation

3 units at 45 degrees

Vx=3 cos45=2.1 units

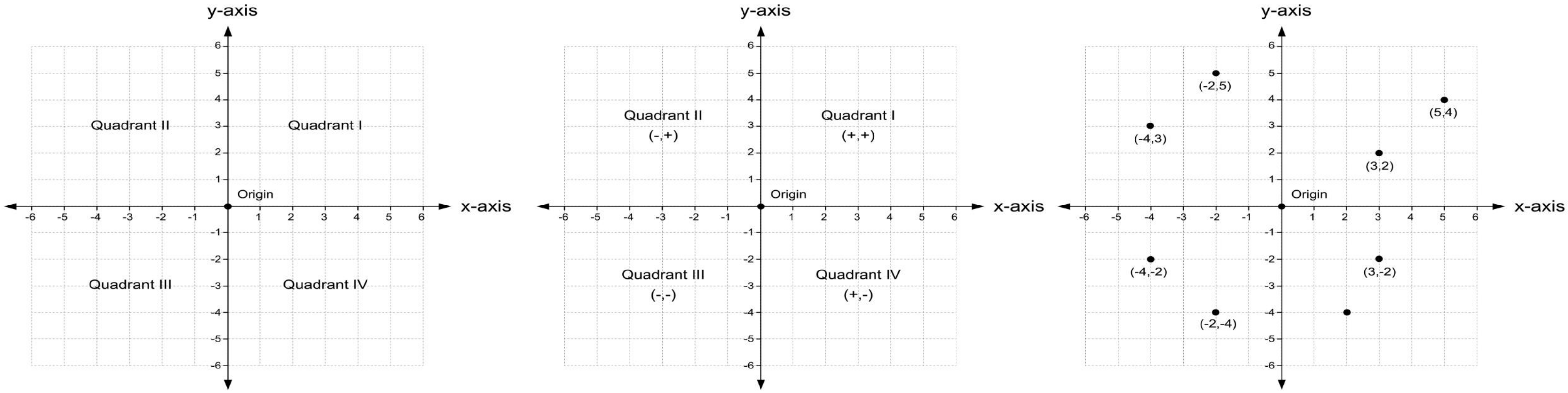
Vy=3 sin45 =2.1 units

V=2.1 i + 2.1 j



QUADRANTS AND DIRECTIONS

First quadrant both components +, second quadrant x component -
third quadrant both components -, fourth quadrant y component -



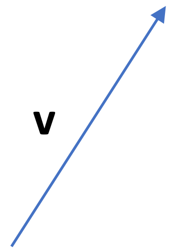
Cartesian coordinate system

MODEL PROBLEMS

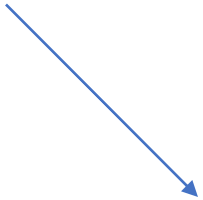
$v_x = v\cos(\theta)$ angle measured with respect to x-axis
$v_y = v\sin(\theta)$ angle measured with respect to x-axis
$v = |\mathbf{v}| = \text{sqrt}(vx^2 + vy^2)$ Pythagorean Theorem
$\tan\theta = vy/vx$
$\theta = \tan\text{-}1(vy/vx)$ or $\theta = \tan\text{-}1(vy/vx) + \pi$ based on quadrant
v= vx **i** + vy **j**

1) x-component of velocity is vx=3m/s and y component is vy=4m/s. Write the velocity as vector representation, in magnitude/angle form, and show it graphically

v= 3 i + 4 j V has a magnitude of 5m/s making 53degrees with the x-axis **v**



2) An object has an acceleration of 4.0 m/s^2 making –45 degrees with the x-axis. Find ax,ay. Show the vector
ax =a cos(θ) ay=a sin(θ) ax=2.3m/s^2 ay=-2.3 m/s^2 **a**



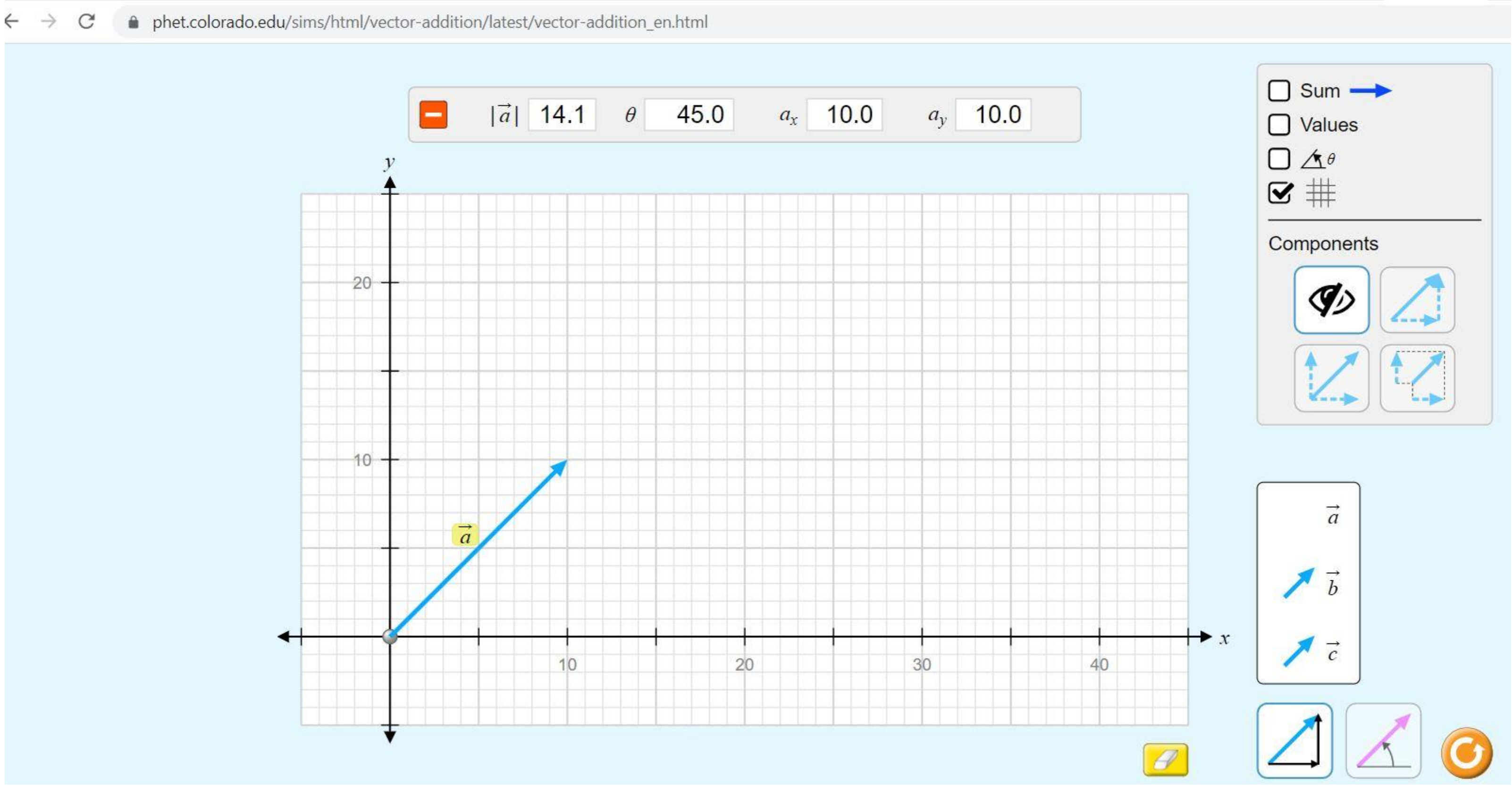
ACTIVITY VECTORS

One representation of an acceleration vector **a** is given. Find the other representations. Choose a suitable scale for the graphical interpretation. For the last problem there are two solutions.

Magnitude Angle Form		**Component Form**		**Vector Form**	**Graphical**
Magnitude	Angle	ax	ay	ax **i** + ay **j**	Choose a scale. Show the arrow in the correct direction
15	53.1				
		-5	-5		
					Two units to the West 
				12**i** + 5**j**	
5		3			

ACTIVITY VECTORS

- Open https://phet.colorado.edu/en/simulations/vector-addition
- Choose explore 2-D. Drag the vector "a" to the cartesian coordinates.
- Explore the sign for the vector components in various quadrants by rotating or resizing the vector.

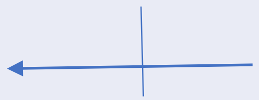


ACTIVITY VECTORS USING PHET

Go to https://phet.colorado.edu/en/simulations/vector-addition and choose explore 2-D

Drag the vector "**a**" and verify your answers

Come up with your own problems and explore

Magnitude Angle Form		Component Form		Vector Form	Graphical
Magnitude	Angle	ax	ay	ax **i** + ay **j**	Choose a scale Show the arrow
15	53.1				
		-5	-5		
					
				12**i** + 5**j**	
5		3			

2-D PROJECTILE
Horizontal x-axis Vertical y-axis

$\Delta y = voy\ t - \frac{1}{2} g\ t^2$

$vfy = voy - gt$

$vfy^2 - voy^2 = -2g\Delta y$

$\Delta x = vox\ t$

$vfx = vox$

Three Cases

A) Horizontal projectile from a height, Voy=0

B) Diagonal projectile from the ground, Δy=0 at impact

C) Diagonal projectile from a height, general case

Model Problem Horizontal Projectile

Q) A piece of rock projected horizontally from a cliff. It stays in the air for 6.0 seconds before hitting the ground and travels a range of 288 meters measured from the bottom of the cliff. Neglect the friction.

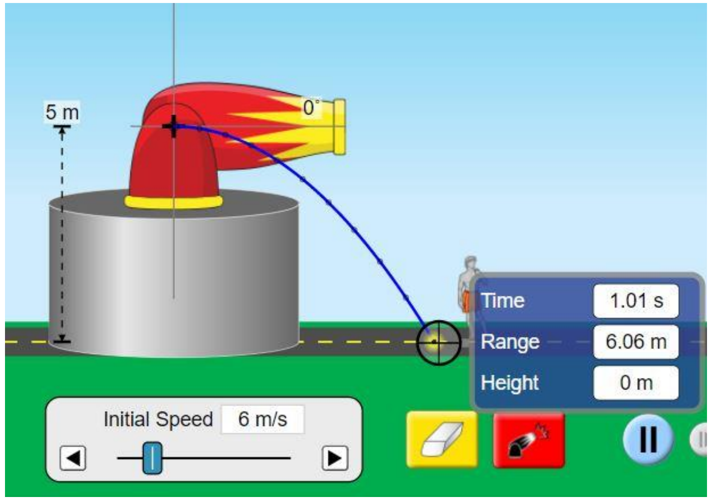
- What was the height it was projected from?
- What was its initial velocity?
- What is y-component of the final velocity at the impact.
- Find the impact speed.

Model Problem Diagonal Projectile

- Q) A golf ball is hit from the ground with an angle of 30 degrees with the x-axis. Golf ball has an initial speed of 10.0 m/s.
- What are the x and y components of initial velocity
- How long will it stay in the air?
- What is the range?
- What is the impact speed?

ACTIVITY 2-D HORIZONTAL PROJECTILE

Open https://phet.colorado.edu/en/simulations/projectile-motion
Drag the cannon up to the initial height
Rotate the cannon to zero degrees, adjust the speed

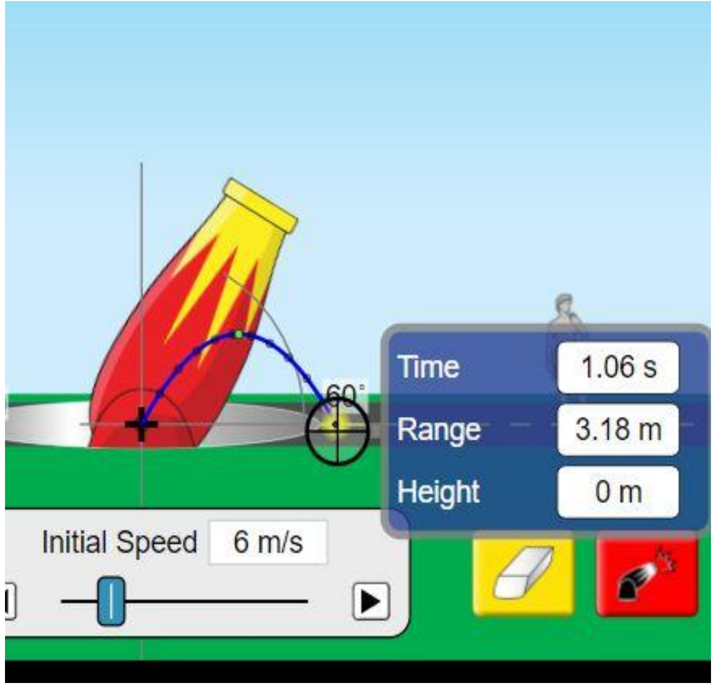


Q1) An object is projected horizontally from 5 meters height with 6m/s speed

- Calculate its location in x and y after 0.2 seconds
- Using the height, calculate time for fall
- Using the time and the initial speed calculate its range
- Verify your values using the simulation
- Repeat the activity for various heights and speed.

ACTIVITY 2-D DIAGONAL PROJECTILE

Open https://phet.colorado.edu/en/simulations/projectile-motion
Drag the cannon up to the initial height
Rotate the cannon to zero degrees, adjust the speed



Q1) An object is projected with 60 degrees angle diagonally from the ground. Initial speed is 6m/s

- When will it reach the maximum height (Vfy=0)
- What is the maximum height
- Find the time for fall and the range
- Verify your values using the simulation
- Repeat the activity for various height, speed and angle.

General 2-D Motion

- For motion on the surface of the Earth, we can take North as +y axis and East as +x axis. We treat the motion as if it is two separate motion

Δx = vox t + ½ ax t^2 vfx=vox + ax t vox = vo cos(θ) ax = a cos(θ)

Δy = voy t + ½ ay t^2 vfy=voy + ay t voy = vo sin(θ) ay = a sin(θ)

Q)A car is initially moving 37 degrees North of East with 20 m/s. It accelerates suddenly towards South with a=4m/s^2. Find ax.ay,vox and voy. Find the displacement in x,y and the final velocity components vfx and vfy in two seconds.

Classwork-Complete the Table

- $\Delta x = v_{ox}\, t + \frac{1}{2}\, a_x\, t^2$ $v_{fx} = v_{ox} + a_x\, t$ $v_{ox} = v_o \cos(\theta)$ $a_x = a \cos(\theta)$
- $\Delta y = v_{oy}\, t + \frac{1}{2}\, a_y\, t^2$ $v_{fy} = v_{oy} + a_y\, t$ $v_{oy} = v_o \sin(\theta)$ $a_y = a \sin(\theta)$

Vo	a	t(s)	deltaX	deltaY	Vfx	Vfy
Vox=4m/s Voy=3m/s	ax=2m/s^2 ay=-1m/s^2	3				
Speed 5m/s angle=30	a= 4 m/s^2 West	4				
6 m/s North	2 m/s^2 East	s				
	ax=2m/s^2 ay=-1m/s^2	6			15 m/s	12 m/s
Vox= 20m/s Voy=15m/s		8	120 meters	140meters		

Come up with your own questions. Make sure they are consistent and sufficient to solve

RELATIVE VELOCITY USING VECTOR ADDITION

A = Ax **i** + Ay **j**

B = Bx **i** + By **j**

A+B = (Ax+Bx) **i** + (Ay + By) **j**

A = 3 **i** – 4 **j**

B = 12 **i** + 14**j**

A+B=15 **i** + 10**j**

RELATIVE VELOCITY USING VECTOR ADDITION

Vab = Velocity of object a with respect to the observer at b

Vbc = Velocity of b with respect to the observer c

Vac = Velocity of object a with respect to the observer c

Vab = -Vba **Vbc = -Vcb** **Vac = -Vca**

For some problems we need to use the formulas above to match index

EQUATION: **Vab + Vbc = Vac**

REPEATED INDEX MUST BE ADJACENT

MODEL PROBLEM

1) Alex is swimming North with 2m/s speed with respect to Big Sur River.

The river is flowing with 3 m/s speed East with respect to Cathy who is observing Alex from the bank. What is the velocity of Alex with respect to Cathy. Vab=Velocity of Alex with respect to Big Sur River. Vbc= Velocity of Big Sur River with respect to Cathy. Vac=Velocity of Alex with respect to Cathy.

Vab = 2i + 0j

Vbc= 0i + 3j

Vac = 2i + 3j

Classwork

1) Vto = 4 i –5j   Vqt = 3i+6 j Vqo=?

2) Velocity of an airplane with respect to the tower is 400mph 45^o Northeast. Velocity of the wind with respect to tower is 30 mph East.

What is the velocity of the airplane with respect to the wind.

3) Vcb = 4j   Vca=3 i –3j   Find Vab

4) A swimmer wants to swim towards North with 2 m/s speed with respect to an observer on the bridge. River runs with 4 m/s towards west with respect to the bridge. What should be velocity of the swimmer with respect to the river

5) Vab= 3i   Vac=5i +6j   Find Vbc and Vcb

REFERENCES

Slide 1: Adobe id=296932788 Projectiles vector illustration. Labeled physical force trajectory scheme. By VectorMine

Slide 13: Adobe id= 171841729 Cartesian coordinate system on white background vector illustration By attaphong

Slide 8-9-10-16-21-22 Screenshot from PhET Interactive Simulations University of Colorado Boulder https://phet.colorado.edu